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Diffusion of a Strong Internal Magnetic Field through the Radiative Envelope of a 2.25 $M_\odot$-star

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Summary. Strong internal magnetic fields of stars are probably relevant for triggering supernova explosions, pulsar and white dwarf magnetic fields and the fields of some magnetic stars. We present numerical calculations for the evolution of a magnetic field produced by turbulent dynamo action in the convective core of a 2.25 $M_\odot$-star. The diffusion of the field through the radiative envelope is investigated.

The field reaches detectable surface values in a time less than 1% of the familiar magnetic timescale $\tau = R^2 \cdot \eta^{-1}$. For stars with $M \lesssim M_c$ ($M_c = 2.25 \ldots 5\ M_\odot$ depending on the core field strength) the diffusion time is less than or at most of the order of the main sequence lifetime.

Consequently, a core dynamo can serve as a model for magnetic stars in the mass range $1.5\ M_\odot \lesssim M \lesssim M_c$. Stars with $1.2\ M_\odot \lesssim M \lesssim 1.5\ M_\odot$ probably possess a field which is a remnant from dynamo action during the Hayashi phase. Stars with $M \gtrsim M_c$ have main sequence lifetimes too short to allow for growing of a detectable surface field. Their hidden fields are required, however, for the production of the observed pulsar magnetic fields.

Key words: stars — magnetic field — stellar structure and evolution — α-effect — dynamo-theory

1. Introduction

According to stellar structure calculations (Iben, 1967; Novotny, 1973, further references given there) stars with a mass $M \gtrsim 1.5\ M_\odot$ possess a convective core.

Recently, Pähler (1976) investigated the possibility and the properties of a turbulent α^2-dynamo working within a convective core of a star by linear analytic calculations. The α-effect describes the dynamo action by means of non-mirror-symmetric turbulence (Steenbeck, Krause and Rädler, 1966; Krause and Steenbeck, 1967). α^2-dynamos are driven by this effect only without support of macroscopic velocity fields like differential rotation.

An internal magnetic field of a presupernova is probably needed as the origin of a pulsar magnetic field or a white dwarf field which is revealed when the stellar envelope is blown off and the core collapses (Imoto and Kanai, 1971; Ruderman and Sutherland, 1973; Levy and Rose, 1974). The subject of the present paper is the question whether a dynamo built magnetic field may become observable already during the main sequence stage. Possible candidates are the magnetic stars belonging to spectral types late B, A, and early F. Magnetic fields of stars with $1.2\ M_\odot \lesssim M \lesssim 1.5\ M_\odot \ldots 2\ M_\odot$ probably are remnants of a dynamo working during the Hayashi phase of the star while it is fully or partly convective (Schüßler, 1975). However, stars with larger masses do not run through a Hayashi phase (Larson, 1972). Their fields may be primeval (Mestel, 1976) or due to dynamo action within a convective core. There may be even a superposition of fields due to both mechanisms.

Gurm and Wentzel (1967) showed that buoyant magnetic "bubbles" can rise through a radiative envelope in a time less than the main sequence lifetime of the star. If this simplified model is correct then mixing would be important for the magnetic stars and the anomalous abundances observed for them could not be explained by accretion or diffusion (Havnes, 1976; Michaud, 1976).

An alternative to the buoyancy model which does not run into difficulties with stellar mixing is the diffusion of the field through the radiative envelope, which is not associated with macroscopic fluid motions. The most important parameter affecting the timescale of diffusion is $q := \eta_r \cdot \eta_T^{-1}$ where η_r is the magnetic diffusivity of the radiative envelope ($\eta_r = c^2 \cdot (4\pi\sigma)^{-1}$, σ = electric conductivity) and η_T the turbulent magnetic diffusivity of the convective core. For a star with $M = 2.25\ M_\odot$ one can estimate $q \approx 3 \cdot 10^{-11}$ (see below). According to the calculations of Pähler (1976), the time needed for the magnetic field to reach a detectable value (say, hundred

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Gauss) at the surface of the star is proportional to $q^{-1/2}$. Inserting numbers one reaches the conclusion that the field diffuses through the star within a few percent of the main sequence lifetime if $M < 10\,M_{\odot}$. These linear calculations have the disadvantage that the field strength in the core reaches unphysically large values during the diffusion. This leads to the suspicion that the field is "pushed" through the envelope by means of the exponentially growing core field which determines the boundary condition at the surface between core and envelope.

The only way to avoid this difficulty is a non-linear calculation including the inhibiting influence of the growing field upon its generation thus limiting the field to a finite value. Though there are some models available for the non-linear interaction of the magnetic field with fluid motions driven by the field (Malkus and Proctor, 1975; Hellmich, 1977; Schüßler, 1977) we use the ad hoc model of the cut-off-α-effect (Stix, 1973) because of its simplicity: The dynamo excitation is cut off, i.e. α is set equal to zero if the field strength exceeds some critical value B_c (e.g. the equipartition value with turbulent motion). Such a crude model is sufficient for our problem because the properties of the field-limiting process do not influence the diffusion of the field outside of the core.

2. Equations

The time behaviour of the field is described by the familiar MHD induction equation which reads in dimensionless variables:

$$\frac{\partial \mathbf{B}}{\partial t} = R_\alpha \nabla \times (\alpha \mathbf{B} + \mathbf{v} \times \mathbf{B}) - \nabla \times (\eta \nabla \times \mathbf{B}) \tag{1}$$

where $\mathbf{B}$ is the mean magnetic field in units of the critical field B_c, $R_\alpha = \alpha_0 \cdot r_k \cdot \eta_T^{-1}$ the magnetic Reynolds number, α_0 the scale of the α-effect with the dimension of a velocity, r_k the radius of the stellar core. We have chosen $R_\alpha = 15$ in order to get an excited dynamo. α and η are dimensionless functions describing the spatial variation of α-effect and magnetic diffusivity. We follow a well-known treatment (see e.g. Schüßler, 1975) to construct the dynamo equations for α^2-dynamos (introducing toroidal and poloidal field, mean velocity $\mathbf{v} = \mathbf{0}$ in a corotating frame):

$$\frac{\partial \mathbf{B}_{\rm tor}}{\partial t} = R_\alpha \nabla \times (\alpha \nabla \times \mathbf{A}) - \nabla \times (\eta \nabla \times \mathbf{B}_{\rm tor}) \tag{2}$$

$$\frac{\partial \mathbf{A}}{\partial t} = R_\alpha \cdot \alpha \cdot \mathbf{B}_{\rm tor} - \eta \nabla \times \nabla \times \mathbf{A}. \tag{3}$$

The vectors $\mathbf{B}_{\rm tor}$ (toroidal field) and $\mathbf{A}$ (vector potential for the poloidal field) have only ϕ-components in a spherical polar coordinate system (r, θ, ϕ). We consider axisymmetric fields only, i.e. $\partial/\partial\phi = 0$ in (2) and (3). The boundary conditions are regularity of the solutions for $r = 0$ and $\mathbf{B}_{\rm tor} = 0$ for $r = R$ (no currents through the surface of the star) and $\mathbf{B}_{\rm pol} = \nabla \times \mathbf{A}$ (poloidal field) has to pass steadily into a potential field outside the sphere with radius R.

The functions α and η are r-dependent only. Smooth fitting at the transition from the convective core ($\alpha \neq 0$, $\eta = \eta_T$) to the radiative envelope ($\alpha = 0, \eta = \eta_r$) is accomplished by using the error function Φ (Roberts and Stix, 1972):

$$\eta(r) = q + (1 - q) \cdot \tilde{\Phi}(r, r_0, d_1) \tag{4}$$

$$\alpha(r, |\mathbf{B}|) = \tilde{\Phi}(r, r_0, d_1) \cdot \tfrac{1}{2}\left[1 - \Phi\left(\frac{|\mathbf{B}| - B_c}{d_2}\right)\right] \tag{5}$$

with

$$\tilde{\Phi}(r, r_0, d_1) := \tfrac{1}{2}\left[1 + \Phi\left(\frac{r - r_0}{d_1}\right)\right] \tag{6}$$

d_1 is the half-width of the transition zone between core and envelope[1] while r_0 is the radius of the center of the transition zone. The calculations use standard values $d_1 = 0.05R$ (the diffusion time does not depend significantly upon this choice), $r_0 = 0.2\,R_{\odot}$ (according to a $2.25\,M_{\odot}$-star), $d_2 = 0.05B_c$. The second factor in (5) was added to represent the cut-off-α-effect with a parameter d_2 accounting for the "sharpness" of cut-off.

3. Numerical Procedure and Parameters

We have to use numerical methods in order to compute $\mathbf{B}$ even if we are just interested in the evolution of the field in the region $r \gtrsim r_0$ where $\alpha = 0$ and the equations seem to be uncoupled. The reason is twofold:

1) We want to describe a non-vanishing transition zone between core and envelope since a sharp transition seems to be unphysical (overshooting, see also Straus, 1976; Graham, 1975; Spradley and Churchill, 1975).

2) If one solves the linear equations in the envelope only one has to use boundary conditions at the core–envelope boundary involving field strength and field gradient. Especially the field gradient within the transition region is responsible for the outward diffusion of the field. Non-linear dynamo action within the core prevents the gradient from being smoothed out as would be the case for a simple diffusion problem.

The numerical method we utilize is the finite difference scheme of DuFort and Frankel (1953) with 40 gridpoints in the r-direction and 12 points in latitude. There are some problems arising from the large difference of the timescales important for the diffusion. The values of η_r and η_T can be estimated as follows.

[1] Schüßler's (1975) q has to be replaced by q^{-1} to be consistent with this paper

The molecular magnetic diffusivity is given by Spitzer and Härm (1953)

$$\eta_r = 519.58 \frac{\ln \Lambda}{T_6^{3/2}} \text{cm}^2 \text{s}^{-1} \qquad (7)$$

$$\Lambda = 16.8 \frac{T_6^{3/2}}{\sqrt{\rho}}$$

where T_6 is the temperature in 10^6 K and ρ the density in g/cm^3.

Taking mean values $\bar{T} \approx 5.3 \cdot 10^6$ K and $\bar{\rho} \approx 6$ g cm^{-3} for the envelope (Novotny, 1973), (7) yields $\eta_r \approx 2 \cdot 10^2$ cm^2 s^{-1}. With $\eta_T = 0.15 \cdot r_k \cdot v_t$ (Parker, 1971), assuming the turbulent velocity to be $v_t = 3 \cdot 10^3$ cm s^{-1}, the turbulent magnetic diffusivity becomes $\eta_T \approx 6 \cdot 10^{12}$ cm^2 s^{-1}. Thus it follows

$$q = \frac{\eta_r}{\eta_T} \approx 3 \cdot 10^{-11}$$

for a 2.25 $M_\odot$-star.

Thus two extremely different timescales determine the magnetic diffusion: the magnetic timescale of the convective core $\tau_T = r_k^2 \cdot \eta_T^{-1} \approx 1$ year and the timescale of the envelope $\tau_r = R^2 \cdot \eta_r^{-1} \approx 10^{12}$ years. The timestep for the numerical scheme must always be a small fraction of the relevant timescale of the problem. Consequently, the timestep should be of the order of days and changes within the envelope would not be seen in a reasonable amount of computer time. To overcome this problem we use a "flip-flop technique": The equations are solved alternating between core and envelope, each with its adequate timestep. The separation between core and envelope is made at some critical gridpoint where α and η are small enough for the equations to be solved with a timestep adequate for the envelope. If the envelope field has changed its size by a prescribed fraction (say, $(|\mathbf{B}_{new}| - |\mathbf{B}_{old}|)/(|\mathbf{B}_{old}|) \geqq 0.1$), the new core field will be calculated according to the new envelope field. With the core timestep the calculation will go on until the core field is stationary again. Then the envelope field will be calculated with its timestep according to the adjusted core field and so on. This iterative procedure is justified since the timescale of the core is short enough for the field to readjust nearly immediately to the envelope solution yielding a quasistatic evolution from the viewpoint of the core.

4. Results

As a first step we calculated a linear model (without the cut-off-effect) for which analytical results are available from Pähler (1976). The only difference between analytical and numerical model results from the finite width of the transition zone for the latter. We found a close similarity of the geometry of the solutions, but the numerical results show different growth rates depending on the width d_1 of the transition zone. For decreasing d_1 the growth rate seems to approach the analytical value ($d_1 = 0$) though d_1 could not be decreased enough to show real convergence without running into numerical instabilities. However, the geometries of the solutions agree and for real stars one should expect overshooting of the convection and therefore a transition zone with an extent of the order of one scale height. This is the situation simulated by our numerical models.

Our standard model has the following parameters: $d_1 = 0.05R$ (transition zone of the order of the core dimension), $d_2 = 0.05B_c$ (quick decay of α for $|\mathbf{B}| > B_c$), $r_0 = 0.2\, R_\odot$ and $R_\alpha = 15$ (excited dynamo). We do not investigate a model with a θ-dependent α (e.g. $\alpha \sim \cos\theta$) because the analytical calculations of Pähler (1976) could not deal with such a model; the numerical model would differ only in dynamo excitation and symmetry properties with respect to the equator. Since the excitation is a free parameter anyhow, it does not seem to be useful to study more complicated models.

The symmetry of the α-effect with respect to the equatorial plane can be used to introduce two kinds of solutions of Equations (2) and (3):

(a) solutions with $\mathbf{B}_{tor}$ and $\mathbf{A}$ symmetric with respect to the equatorial plane (therefore $\mathbf{B}_{pol} = \nabla \times \mathbf{A}$ antisymmetric): "dipole parity";

(b) solutions with $\mathbf{B}_{tor}$ and $\mathbf{A}$ antisymmetric (therefore $\mathbf{B}_{pol}$ symmetric): "quadrupole parity".

We shall only show results for models with dipole parity because they exhibit a dipole-like field in the exterior of the star. The properties of the solutions with quadrupole parity, especially those concerning the diffusion time, are quite similar to those with dipole parity.

A model with $q = 3 \cdot 10^{-11}$ could not be calculated for numerical reasons (instabilities etc.). We present a series of models with $q = 10^{-3}, 10^{-4}, 10^{-5}$ and 10^{-6} and try to extrapolate their behaviour to a model with $q = 3 \cdot 10^{-11}$ corresponding to a 2.25 $M_\odot$-star. Let us estimate the critical magnetic field B_c because the maximum field produced by the dynamo will reach this order of magnitude. We suppose equipartition between magnetic energy density and turbulent kinetic energy density for the critical field:

$$\frac{B_c^2}{8\pi} = \tfrac{1}{2}\rho u^2. \qquad (8)$$

Taking core values of $\rho \approx 60$ g cm^{-3} and $u \approx 30$ m s^{-1} the critical field is of the order of $B_c \approx 10^5$ Gauss. We shall call a surface field of 100 Gauss, i.e. $B = B_c/1000$ "detectable" and give the time it takes to reach such poloidal field strength at the surface. We also give the time it takes for the toroidal field to reach 100 Gauss at

Table 1. Diffusion time until a value of 100 Gauss is reached at the surface (poloidal field) and one grid point below the surface (toroidal field), respectively, for different values of $q = \eta_r \cdot \eta_T^{-1}$

q	t_{pol}/τ_T	t_{tor}/τ_T
10^{-3}	50	16
10^{-4}	900	250
10^{-5}	5300	2500
10^{-6}	slower by a factor 10 than for $q = 10^{-5}$	

the first grid point below the surface. We shall call these the "diffusion times". We measure the time in units of the magnetic time scale of the core $\tau_T = r_K^2 \eta_T^{-1}$ which is of the order of one year. This is the time required by the dynamo to build up a stationary field from some infinitesimal "seed field". Table 1 gives the diffusion time for the poloidal and toroidal field, respectively, for different values of q.

For $q = 10^{-6}$ the model was not computed until a detectable value was reached because of the slow evolution of the field. The velocity of the 100 Gauss surface moving outwards is 10 times smaller than for the model with $q = 10^{-5}$. Consequently, it seems to be established that the diffusion time for poloidal and toroidal field is roughly proportional to q^{-1} in contrast to a proportionality to $q^{-1/2}$ proposed by linear theory (Pähler, 1976). A test calculation with $q = 10^{-8}$ could not be carried out far enough to get a certain result but seems to support this conclusion. The value of 900 τ_T for $q = 10^{-4}$ deviates from this picture by a factor 2 as the only example in the range $q = 10^{-3} \cdots 10^{-8}$. *Roughly* we can conclude a proportionality of diffusion time with q^{-1}. This is also supported by comparison of the rising velocity of the region with $|\mathbf{B}| = 100$ Gauss for different values of q.

Figure 1 shows the time evolution of the envelope field configuration for $q = 10^{-5}$. Lines of equal toroidal field are drawn on the left-hand side of the half circles while field lines of the poloidal field appear on the right-hand side. The outermost contour of the toroidal field refers to a field strength of 10 400 Gauss, with field strength increasing by about 1200 Gauss at each following contour. The strong core parts of the poloidal and toroidal field are omitted in order to show only the moving weak envelope field. The center of the transition zone between core and envelope is drawn as a small half-circle. Figure 1a represents the starting point of the evolution: the core field has developed to a maximum field strength of about 10^5 Gauss. The following figures (Figs. 1b–h) show the further evolution. The toroidal field stays nearly stationary after about $5000\tau_T$ while the poloidal field is growing slowly even after $13\,000\tau_T$. The faster evolution of the toroidal field is caused by the term $1/r \cdot \partial\eta/\partial r \cdot \partial/\partial r(B \cdot r)$ in Equation (2) which does not appear in Equation (3). Hence, the non-vanishing gradient of η in the transition zone is responsible for the faster evolution of the toroidal field. B is "driven" by $\partial\eta/\partial r$ and then "pulls" A on reasons of non-linear coupling within the transition zone. At the same time new field is produced within the transition zone preventing the field gradient from being smoothed out by diffusion. Figures 2 and 3 give profiles of poloidal and toroidal field strength, respectively, depending on the radial coordinate for $\theta = 90°$ (equator). They refer to the field configuration of Figure 1h. The toroidal field is negative ($\triangleq$ reversed) within the envelope. This property is caused by the η-gradient too because it yields a negative contribution to the left side of (2) outside the core. The secondary maximum of $|\mathbf{B}_{pol}|$ for $r \approx 0.3$ results from the sharp decrease of $|\mathbf{B}_{tor}|$ in this region yielding a steep gradient $\partial A/\partial r$. Outside of the transition zone the decay of the field is nearly exponential.

The diffusion time is in all cases significantly smaller than the familiar magnetic timescale of the envelope

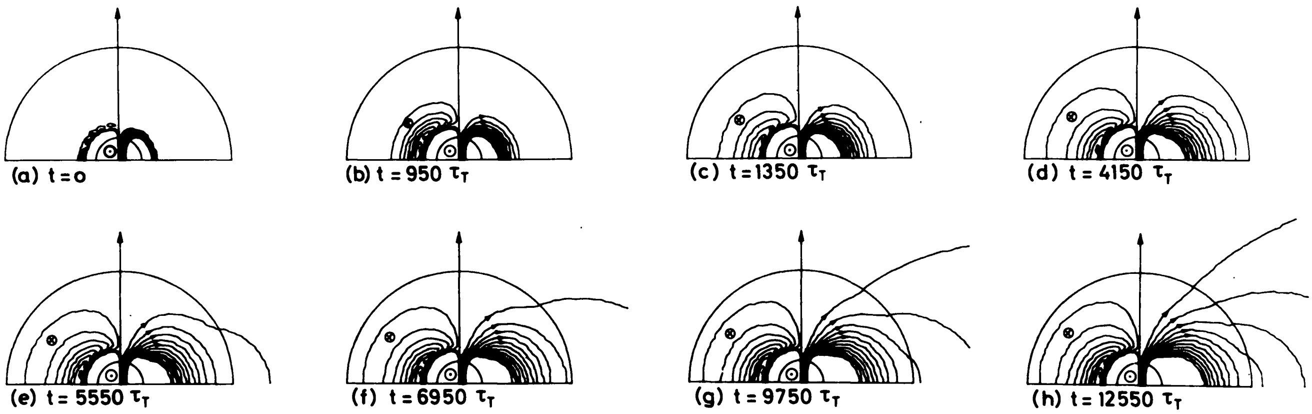

Fig. 1. Envelope field configuration at different moments of the evolution for the $q = 10^{-5}$ model. Left-hand side of half circles: Lines of constant toroidal field (direction of the field lines indicated). Right-hand side of half circles: Field lines of the poloidal field. The core part of the field is omitted

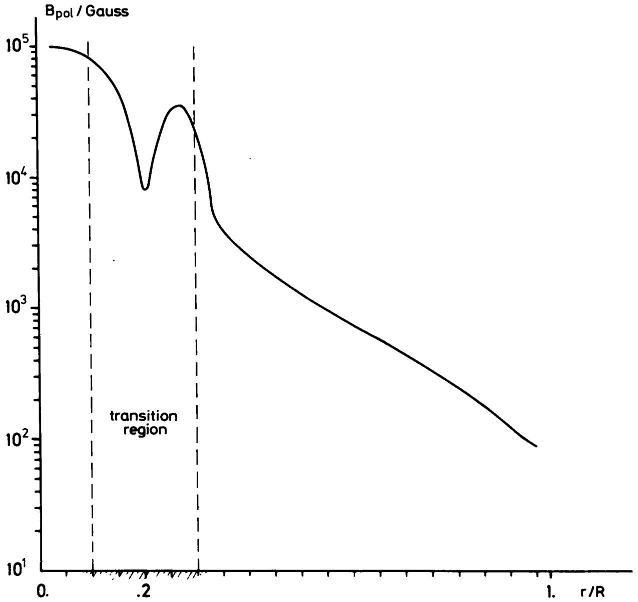

Fig. 2. Poloidal field strength as a function of r/R for $\theta = 90°$ (equator)

$\tau_r = R^2\eta_r^{-1} = R^2\eta_T^{-1}q^{-1} \approx 50y \cdot q^{-1}$ with $R = 1.45\,R_\odot$ for a 2.25 $M_\odot$-star. Table 2 gives the ratios of diffusion time to magnetic time scale for different values of q for toroidal and poloidal field, respectively. One can see that the field is able to diffuse through the envelope much faster than it is predicted by a simple timescale estimate. This conclusion is only correct if the "detectable" field of 100 Gauss does not differ too much from the final surface value reached by the field strength for a steady

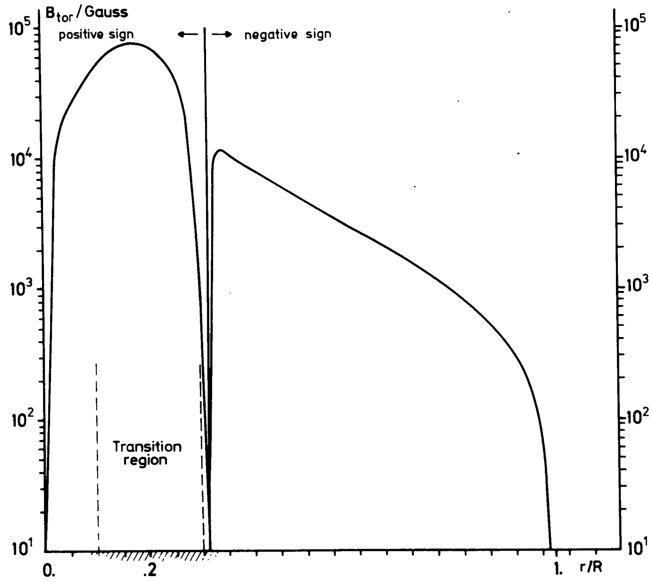

Fig. 3. Toroidal field as a function of r/R for $\theta = 90°$ (equator). The logarithmic scale requires reversed drawing of the part with negative sign

Table 2. Ratio of diffusion time to magnetic timescale for different values of $q = \eta_r \cdot \eta_T^{-1}$

q	t_{pol}/τ_r	t_{tor}/τ_r
10^{-3}	0.0010	0.0003
10^{-4}	0.0018	0.0005
10^{-5}	0.0011	0.0005

state. For simple diffusion τ_r measures the time taken by the surface field to reach nearly its final value. Our calculations show indeed that detectable and final field agree to order of magnitude, i.e. the final field is of the order of tenths of a percent of the core field. This leads to the suspicion that the equipartition estimate for the core field yields values too small to account for observed fields of 1000 or even 30 000 Gauss. Increasing B_c by one order of magnitude as indicated by the considerations above would decrease the time until 100 Gauss surface fields are reached by the same proportion.

In any case, simple diffusion as described by τ_r is not a relevant model for our results which seem to reflect a forcing mechanism due to penetrative convection. The reason for this behaviour is the dynamo action within the core inhibiting smoothing out of the field gradient in the transition zone. This pushes the field through the envelope much faster than simple diffusion without sources would do. However, the diffusion time is significantly larger than that predicted by Pähler's linear theory, showing the relevance of the field limitation process.

But is this forced diffusion fast enough to cross the envelope within the main sequence lifetime of the star? Assuming the proportionality with q^{-1} suggested by the calculations, the diffusion time for $q = 3 \cdot 10^{-11}$ can be extrapolated from the above results: $t_{pol} = 5300 \cdot (10^{-5}/3 \cdot 10^{-11})$ years $\approx 1.8 \cdot 10^9$ years which is the order of magnitude of the main sequence lifetime of about $1.2 \cdot 10^9$ years. Thus, for an equipartition core field, the 2.25 $M_\odot$-star seems to represent a borderline case: More massive stars have shorter main sequence lifetimes leaving not enough time for diffusion of the field while stars with a smaller mass live longer and we should expect diffusion of the core field until observable values are reached. Assuming a core field ten times stronger than the equipartition value (see above) even stars with 5 $M_\odot$ can show a field beyond the limit of 100 Gauss within their main sequence lifetime. However, until we have detailed non-linear calculations of the core dynamo the borderline mass cannot be given without speculation. On the other hand, equipartition is no "dogma" as indicated by Galloway et al. (1977), Busse (1975).

5. Conclusion

The results of numerical calculations show that a magnetic field produced by dynamo action in the convective core of a star can diffuse through the radiative envelope to reach detectable values much faster than it is predicted by the ordinary magnetic timescale. The results suggest a proportionality of the diffusion time to q^{-1} in contrast to the linear results of Pähler (1976) inferring a diffusion time proportional to $q^{-1/2}$. This illuminates the importance of non-linear calculations which do not exhibit unlimited growing of the core field, thus leading to wrong conclusions. Another point is the strong subsurface toroidal field produced by diffusion (see Fig. 3). Reaching levels with small enough scale height it may grow unstable and emerge through the surface (Gilman, 1970). This can be relevant for complex field geometries and/or irregular variables.

For $M \lesssim M_c$ ($M_c = 2.25 \cdots 5\ M_\odot$) extrapolation leads to a diffusion time less than, or at least of the order of, the main sequence lifetime. Consequently, the fields of magnetic stars in the range $1.5\ M_\odot \lesssim M \lesssim M_c$ may originate from an active dynamo in the stellar core while fields of stars with $1.2\ M_\odot \lesssim M \lesssim 1.5\ M_\odot$ (possessing no convective core) probably are remnants of dynamo action during the Hayashi phase.

Stars with $M \gtrsim M_c$ do not spend enough time on the main sequence for the core field to reach detectable surface values. However, strong magnetic fields in the cores of massive stars may be relevant for triggering supernova explosions and pulsar or white dwarf magnetic fields (Imoto and Kanai, 1971; Ruderman and Sutherland, 1973).

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